An Invitation to Category Theory for Designers

Edmund Harriss and Rhett Gayle

Abstract

Abstract systems such as the drawing, musical notation and weaving patterns are not new to art and design. Yet, with the introduction of computers, the range of possible abstractions and the relationship between them is only growing. The mathematical tools developed in category theory, the study of abstract spaces and the maps between them, give a powerful tool to map abstract systems and systems of abstract systems, and can be applied to systems as diverse as ninth century religious imagery and modern architectural form. At a more detailed level it provides a design metaphor for CAM software and visual programming environments such as Grasshopper.

This paper introduces the fundamental concepts of category theory and then shows how they can be applied to a wide variety of situations from design to the wider world.

Introduction

Metaphor and abstraction have been at the heart of art since its beginning, certainly long before the specific notion of abstract art. They can also be seen as central to the study of mathematics, with a reading of the subject as the precise study of a family of metaphors. That precision has enabled mathematicians to dive deep into abstraction building metaphors that help to explain metaphors. One of the deepest subjects within this study is the notion of Category theory [MacLane10], a consideration of abstract spaces and the ways that you can move between them. At its heart is a simple visual metaphor, objects connected by arrows. The objects are considered to be spaces with structure and the arrows give maps taking elements of one space into another. As a key insight the objects under consideration are the spaces themselves, not the specific items they might contain.

In the spirit of mathematics as metaphor [Manin07], we wish to recast Category theory here, as a game, a way to think about structures of ideas, and process. The role of mathematics considered is essentially one of perception [HSC19], an attempt to draw out hidden structure and metaphor, or to create metaphor with an intention of learning. The test for success, therefore, is not the depth, or even correctness, of the mathematical thinking but the question of whether new and useful insights are created into the idea, object, problem, or aesthetic under consideration. Within this framework we will consider both conceptual and more practical processes, with a particular focus on the consideration of how computer representations and algorithms can be brought in to an artistic flow.

We will only look at the surface of the subject from a mathematicians standpoint, but hopefully show enough to inspire ideas or even a deeper study[LS98,Badiou14]. To reiterate, the goal is not to push towards rigorous models but to inspire metaphor, the test for success is whether in a given situation the ideas discussed here provide insight.

The value of abstraction

Begin with a simple exercise. Take a look around and find some objects you could count. They can be simple or complex. Find some things you can give a number to just by looking, some things you would have to count easily and some things that you could count but it might take a while.

You have just engaged in a simple mathematical model of aspects of the world around you, that might be summed up as follows.

Collections of objects ⟶ Counting Numbers

This is our first arrow to interrogate. A couple of key questions immediately come to mind, it is clear that some information is retained, but also much is lost. In order to count you had to take different objects and identify something about them that was the same. Books are a great example. Each is filled with distinct knowledge and stories, but to count them I only have to think of them as books, that information is lost in the map. So for any arrow that we draw there is always a question, to consider both what structure is retained as well as being aware of what structure is lost.

A great example, from architecture, is the drawing. The abstract space of drawing allows ideas to be explored. A drawing represents enough of the structure of a building that ideas can be tested out quickly and inexpensively. After the form of a building is decided upon the drawings play a further role, communicating the design to others. Models provide a different representation retaining more of the three dimensional structure. The emphasis on the architect as author, not constructor, show the importance of abstraction in the ideas of architecture, at least from Leon Battista Alberti's *De re aedificatoria,* in the fifteenth century, onwards [Alberti88, Carpo11].

At this point it is worth noting one rule that we want to apply to our arrows. As we describe what they do, for any input we need some sense of a unique output. If we really want rigour (or are working on a computer) then this is a strong requirement, but the world of metaphor does give some wiggle room. The correct arrow here is therefore not from drawing to building, a single drawing could produce buildings multiple times. The opposite arrow, from building to drawing captures the sense better. If we were to define carefully the rules for a drawing a building could be captured. So the arrow we want to consider here is:

Drawings of Buildings ⟵ Potential Buildings

Many other abstractions have been used within crafts for the length of human existence: musical scores and dance notation [Taruskin09], knitting and weaving patterns [Albers65,Nargi11], the stick models used by ancient pacific navigators to show wave patterns [Lewis94], even language itself. In each case these abstract models provide a similar collection of benefits. They make it possible to repeat a situation, possibly with changes and permutations. They help divide up a problem, cutting away less important properties, making it easier to focus on different aspects independently. Finally they enable that simpler space to be explored more efficiently, enabling new discoveries.

The computer introduces many new worlds of abstraction. More importantly it provides an automated way of moving between abstract spaces. A 3d model of a building opens up many possibilities. It can produce detailed drawings that can be printed out and, with 3d printing produce physical models. With the right information added, it can give running totals of materials needed and overall cost. It can even be made parametric, changing shape based on numbers, such as the size of the plot, or even more complicated inputs such as curves or photographs. A single system can therefore produce an incredible array of designs for different buildings, which can be turned automatically into plans and models.

In any consideration of abstraction, the computer should just be considered as a new tool. It is, however, a tool that makes abstraction itself more powerful. With a computer the process of making choices and following abstract rules can be done automatically. This enables logical processing of an abstract model, rather than simply representation. The primary model has moved from the raw representation of alphabet and drawing to the algorithm [Carpo11]. This enables the faster processing of abstract systems that can be described on the computer. It also makes it far easier to create new abstract worlds and explore their potentials.

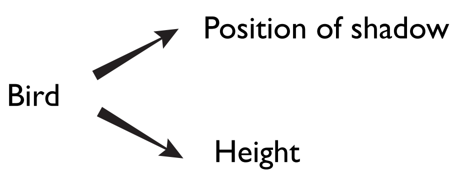
Mathematics faced a similar situation in the nineteenth century as it started to study abstraction itself, rather than simply abstractions from the world [Grey08]. The solution was to apply abstract techniques to abstraction. The parallel for designers is to apply design thinking to the process of design, making it possible to understand skills already developed so that they can be applied to the new possibilities opened up by computers.

Abstraction of abstraction

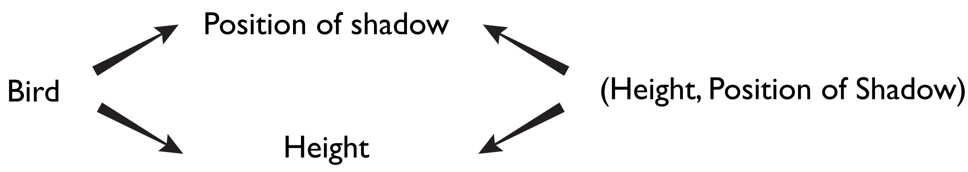
Now consider a sunny day at noon on the equator (so the sun is directly overhead). In the sky a plane is flying and casting a shadow on the ground. The shadow moves with the plane, so we have a new arrow.

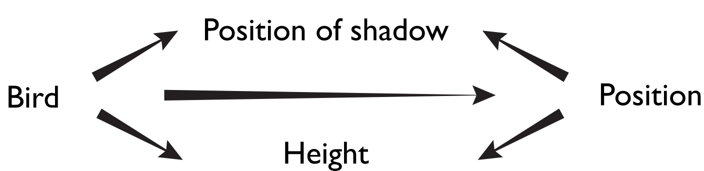
The position of the shadow gives some information about the bird. Yet we cane explicit about something that is missing. Two birds can have the same shadow. We can bring in a second map.

The fact that the same thing (bird) appears in both maps means that we can combine them into a single picture.



The goal here is to create a diagram and then use that to ask questions. In this case we have explicitly identified information that was lost in finding a second map, so we can try to combine the two pieces, shadow and height. Formally we call this a product. From this we magically produce a new space (height, shadow position) from which we can pull out either element.



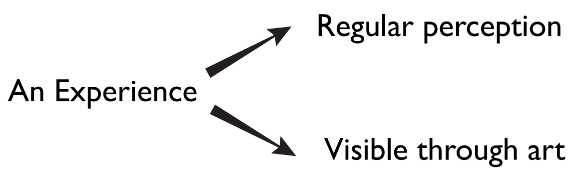


We can now ask if we can describe this new combined space, and perhaps fill in some of the missing arrows. In this case the combined information about shadow and height describes the position of the bird, and we can create a new arrow linking bird to this position.
Combining the two descriptions of the bird gives a richer understanding, yet it is still missing a lot of information. For example the direction of the bird, or its species. Depending on what we wish to study these might be useful or not. In this case the bird is simply a metaphor itself, creating a diagram.

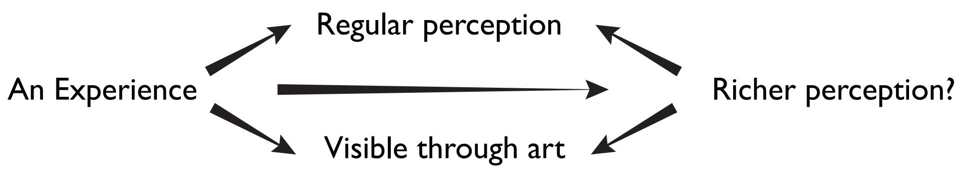
The same diagram can be applied to many different things. For example take Paul Klee's famous notion

"Art does not reproduce the visible; rather it makes visible"

This creates the same starting frame:



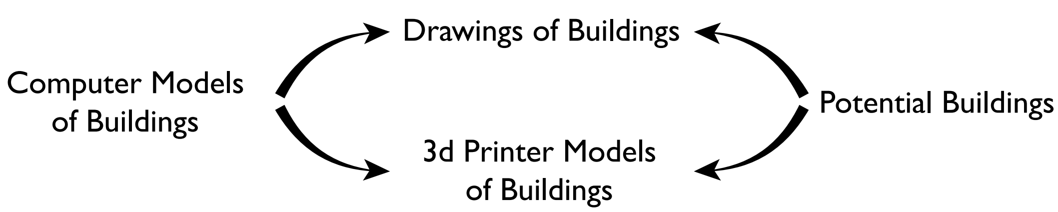
As soon as we have this frame we can ask if it can be extended in the same way. Giving:



With the natural (and possibly interesting) questions of what the richer perception entails and how it informs the original experience.

So to consider abstraction itself we begin with the very loose idea of a set or space, some collection of objects that can be described. The set of all possible houses for example, or the space of all drawings of houses. The second notion that we need is notion of the arrow discussed above. We can return to the process of building (or simply imagining) a possible house from some drawings. Recall that one of the strengths of abstraction is that it helps us to concentrate only on the important properties of the system we wish to study. In this case, therefore, we can ignore the fact that the translation between drawing and building is a difficult one obtained through several years study and just consider that it exists, being summed up in the following diagram:

This is very simple, but with the computer we start to get into more complicated situations. For example with a computer model, we can automatically create both drawings and 3d printed models.



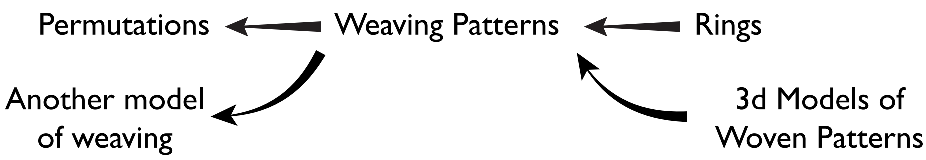
This produces a diagram not that different to the one above. In this case though we have started with two different spaces each loosing information as they map down to two representations. Can we add any reasonable arrows to the diagram? A single computer model, like a drawing can be used to create multiple drawings, but it

In this case each map is simply between spaces, so we might want to consider, in addition, whether the potential house described by the drawing and the 3d model are the same. We might also consider if the potential house that the client translates from the model and the drawings is the same as the one the architect is considering.

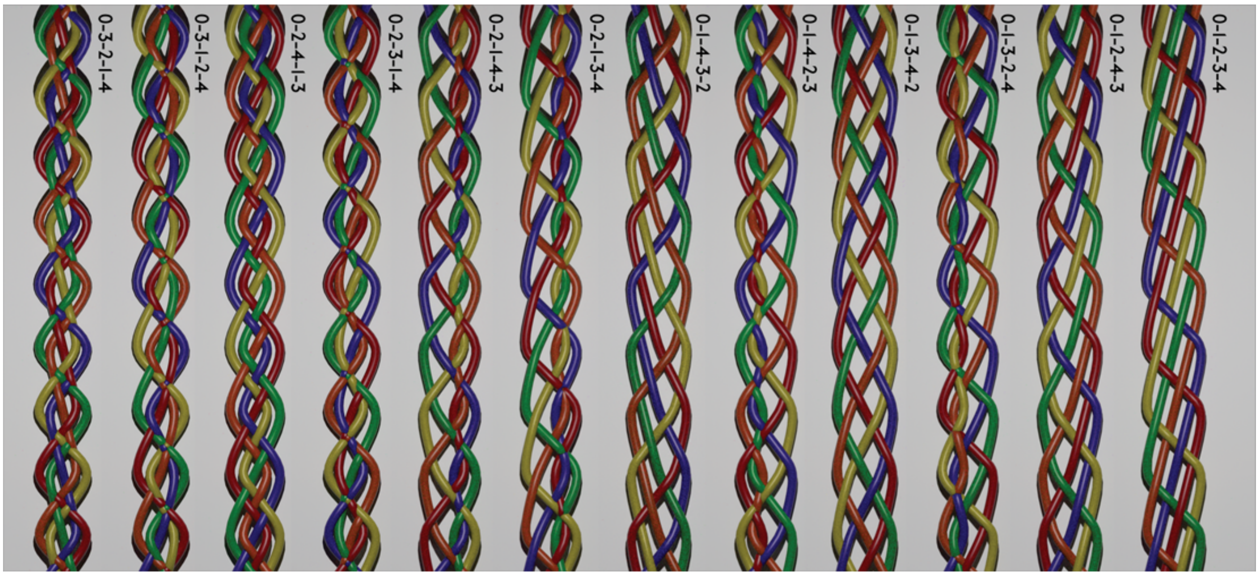
Here is a second example that demonstrates the values of separation of problems, reusability and exploration possible in an abstract system. When the first author was designing his wedding rings he took a weaving pattern he had designed on the computer and wove it in copper wire. This was wrapped round a blank, and the over-cuts filled with epoxy so he could make a mould and sand-cast the rings. Finally they were polished and finished.

A simple process that could be repeated. In fact it can be repeated not just with one pattern but with any weaving pattern you could design. The computer scripts used to design the weaving pattern also had an input space, mathematical permutations, from which they produced weaving patterns. We can consider the process like this:

In this case the operations are a lot more concrete than those discussed above. Yet they still needed some effort to develop. By considering this little model, however, we end up with a significantly more flexible system. The input permutation can be changed without incurring new development costs (there would of course still be some manufacturing costs). A significant aspect of the final design of the rings can therefore be left to the end of the process. Additionally, as the two methods are split, it would be possible to introduce a more sophisticated method of creating weaving patterns. These could then be turned into rings reusing part of this process. Similarly the script to create weaving patterns could be used in other ways.



For example a script could turn the weaving patterns into three dimensional models, allowing a wide range of different patterns to be quickly explored.



A collection of weaving patterns created by an automatic process from mathematical permutations.

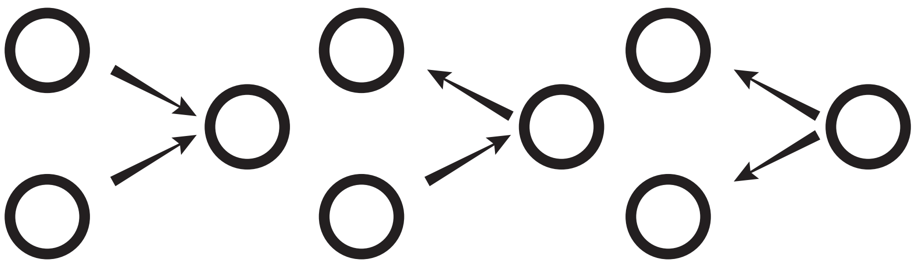
The abstract approach breaks the problem into pieces and reveals potential for new development. Category theory provides a general approach to such thinking.

The Category Theory Game

The heart of category theory rests on three simple ideas:

- Spaces, considered as objects in their own right
- Maps between spaces
- The structures preserved by the maps.

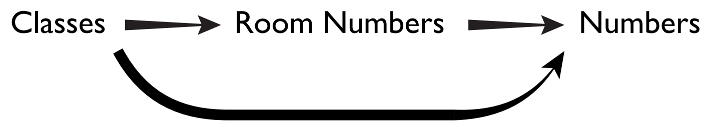
The real object though is just the objects (dots, or words) and the arrows between them. For example with three objects an two arrows there are three different objects to draw:



If the objects were distinct you would have a choice of how they fitted into the three dots, of course. With these drawings in hand for the simple set up of your ideas and their relationships you can now ask questions, for example if two collections have the same structure what useful questions form one can transfer to the other? Another standard question is to ask what arrows are missing. To get to that we need to think a bit more carefully about the arrows.

Each arrow is defined by three things, the *domain* the space it maps from, the *codomain*, the space it takes elements to and some rule or method that for each element of the domain gives an element of the codomain. As described above this rule should aim to give a unique and consistent output for any input. A natural arrow is to describe some process, but if that is repeatable, the natural rule is backwards, taking a product back to the generating plan.

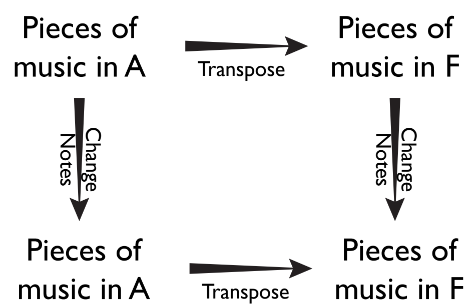
Once arrows are defined the question to ask is about the arrows that are missing. In the triangles above the arrows possible from the top to the bottom dots. The central image is perhaps the simplest. Arrows can be naturally combined. If the codomain of one map is the same as the domain of another then we can create a third map. For example if we have a map from classes to room numbers and another map from room numbers to the number of people in a class, we can combine the two to get a new map giving the number of people in a class:



So any time an arrow comes in and another one goes out they can be combined. This creates two routes between one space and another. In this case by definition the two routes give the same result. If all routes through arrows around the diagram give the same result we say that the diagram *commutes*.

The third concept is perhaps the most important. The spaces under consideration have some sort of structure. The most interesting maps are those that preserve all or some of that structure. A simple example would be transposing a piece of music. Although the resulting piece is certainly distinct it retains all of the structure of the original. Architectural drawings have a similar property, though, in this case, when we apply the map (from the space of possible buildings) only some structure is retained. The value of the drawings is that the structure that is retained is of great importance.

In both these examples the notion of preserving structure goes further. Operations in the domain correspond to operations in the codomain. For example, changing notes in a piece of music corresponds to a similar change in its transposed version. Consider the following diagram:



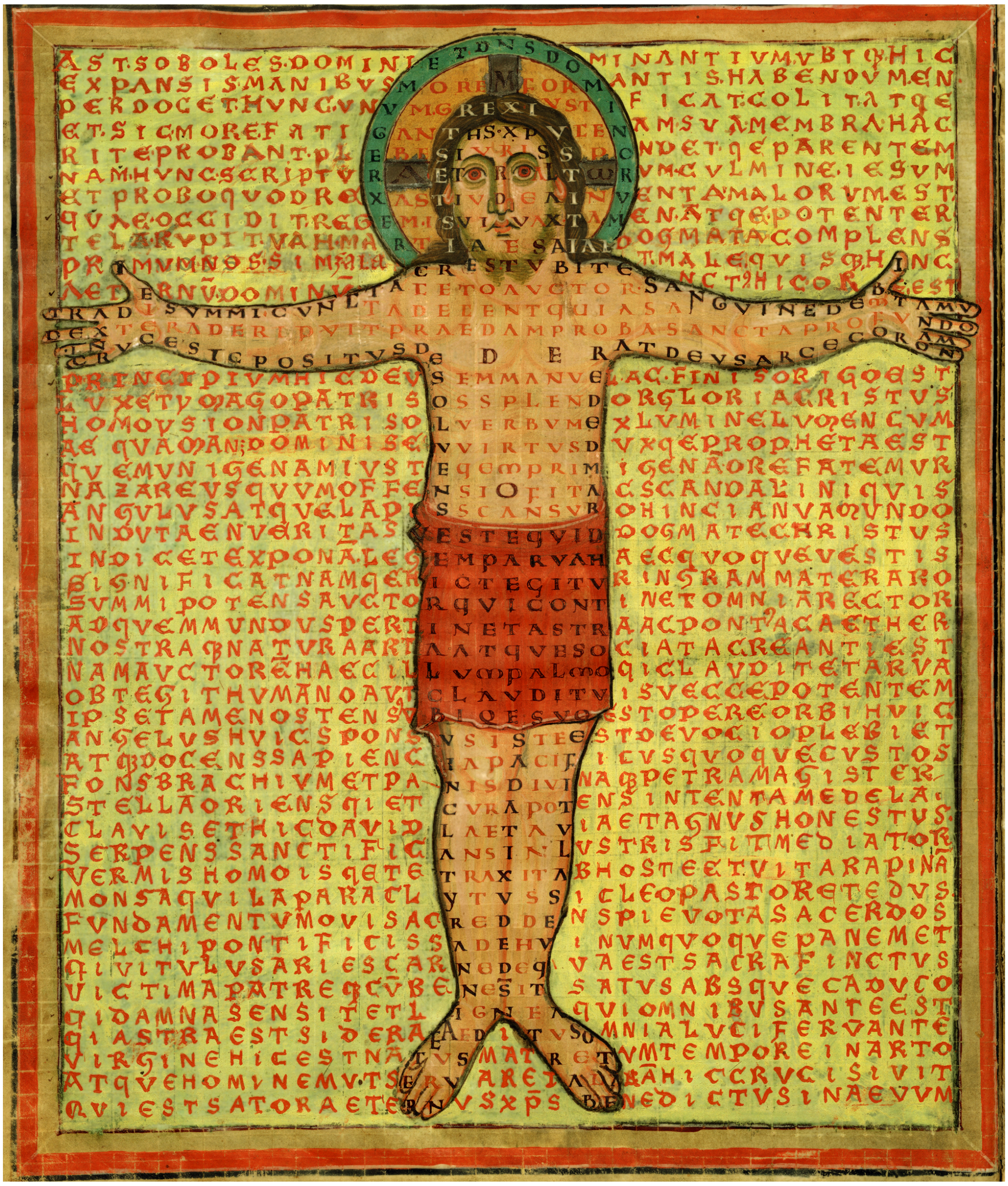
The operations to change notes in the key of A and the key of F will be different, but the same operation applied to two pieces in A will correspond to the same operation on the transposed pieces. In addition, starting with a piece of music in the top left and following either of the two paths will result in the same piece of music in the bottom right.

In cases where the map clearly loses some structure, for example, when we map from a performance of a piece of music to the score used. The performance introduces many factors that are not present in the score, yet changes in the performance still correspond directly to changes in the score. The same is true of drawing for designers, it is the means to represent enough of the structure of an idea that it can be communicated to someone else. This is only possible as the map from the ideas in one's head to the drawings retains the right amount of structure.

The majority of maps and translations that we use intuitively in life do have this property of retaining some or all structure. It can be easy to assume that this is always true, that just because we have a map, it retains structure. It is therefore important to consider the structural properties independently of the rules to construct the map.

For the discussion above to really be mathematically correct there is a need to be very careful with arrows, and their properties. This introduces a different collection of interesting games, ensuring that every element of the domain is really connected to a unique thing and also the questions around how arrows can be reversed.

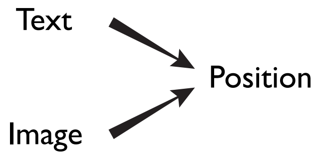
Category theory in action



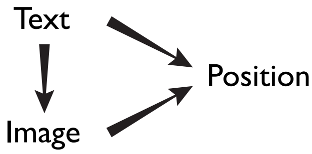
[Maurus830], Image ©Vatican Library

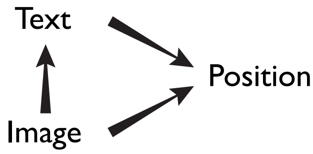
This image comes from the court of Charlemagne, it shows Jesus on the cross, entangled with a collection of poems and comes from *In honorem sanctae crucis,* by Hrabanus Maurus, from around 830

AD [Maurus830]. A gentle sense of mapping comes from two arrows, from text to position and from image to position:



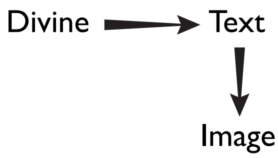
This triangle does not offer any additional arrows by combining as position is the end point of both, yet the third arrow is an obvious question, even having two options:

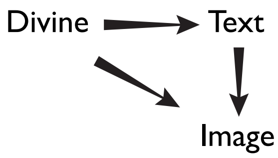




In both of these diagrams the new arrow could combine with one of the older ones to produce an arrow between two spaces. To have a diagram that commutes we need that arrow to be the same. In other words (taking the left hand version) going from text to image, and then to position, should always give the same position as going directly from text to position. In other words, noting that position and text exist together causes a natural question of the link between the text and image at different positions. The dark words around the edge of the body strengthen this mapping, and themselves form sub-poems even as the letters are part of the longer poem.

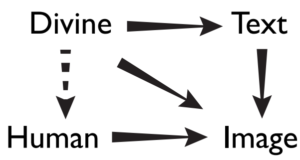
The language and imagery itself evokes other concepts, in particular (and unsurprisingly given the image content) notions of the human and the divine. In a central triangle on the image, black letters spell out "DEO" a clear arrow from the divine to text:





This arrow combines with the one we formed above to link a notion of the divine to positions on the image. Those three positions, however lie on the nipples and navel of the figure, some of the most human aspects of the body. In fact it is unclear if the navel O is text or image. These are thus positions that are part of the arrow:

Combining the two diagrams gives:



With a more subtle question. Is there an arrow that maps from the divine to the human that will combine with the arrow from human to image to give the same result as our route from the divine, to text and then to image? This question, coming simply from identifying and combining arrows takes us from the image to one of the central meditations of Christianity, and certainly towards the artists intent in creating the image. This image and text become a puzzle to locate and relate the other links, ideas and questions. Looking at each individually might not be hard, but one can become lost in the abstractions. Diagramming, provides a tool to think about and discover other more subtle links, to see commonalities between links (as the same diagram repeats) and more [Coon16].

computer aided manufacturing (CAM). The simplest map that can be considered here is the map from the space of physical objects to the space of machine movements. The map takes in some object and gives a list of instructions to the machine that will create that object. For most purposes this is the most practical approach and indeed is the set up used for most CAM software. When approaching and starting to use a machine, however, many people have a different thought: "How do I make the machine make this movement", demonstrated by a hand gesture. This sort of control is not possible with this single map as, instead it requires a new space, the space of geometric toolpaths on the computer. This space gives a very natural representation of the geometric movements that the machine will make, giving a very good representation. The original map can be recreated as a map from physical objects to toolpaths. The map from toolpaths to machine instructions can then be used to generate the machine instructions:

The new intermediary space has the benefit of making the motion of the machine transparent as it retains the actual movement of the machine, rather than just the geometry of the finished object. In addition it introduces the possibility to create new uses for the machine by replacing the map from physical object to toolpath. For example the shape of the path can be used to give patterning on the surface of the finished object[1].

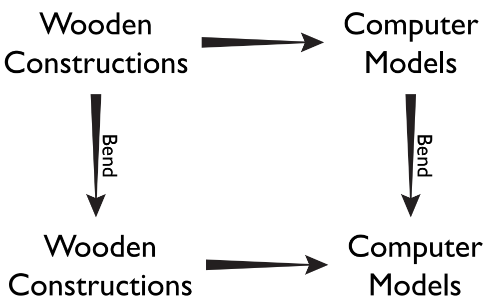
A less concrete example of the application of category theory comes from biomimesis [W-T45], taking maps from the forms of nature onto architectural forms. Classically this has been achieved for static forms, such as the treelike support structures of the Sagrada Familia in Barcelona. To really understand and mimic the process, however, we need to go beyond the forms and start to explore the algorithms used by nature. We can make forms that grow like a tree rather than simply looking like one. We map the pattern of growth from a natural process onto an algorithm that can be run by hand or on a computer [PL91]. Understanding this mapping and the structure it preserves then enables the designer to step in. Not just to use the system to generate a collection of possibilities to select from but to control the process and find curious and beautiful examples.

Such possibilities are developed in the work of Achim Menges. He considers the form finding properties of natural materials like wood and creates a computer model that mimics the important structure. This creates two spaces, the material constructions and the computer model, with a map between them. The computer model enables an easy exploration of the design space, but many of the rules that it is following copy those of the material. As a result the construction does not require complicated work as the rules to follow are already embedded in the materials being used [Menges12]. A notable example is in his bent wood pavilion [FKLMS12].

[1] This discussion comes from the first author's design for a CAM system, being implemented as a Grasshopper plugin (named CAMel).

Images of Achim Menges' Bent wood structure and computer model. The systems are set up so that any bend in one corresponds to a bend in the other.
From http://www.achimmenges.net/?p=4443 © Achim Menges

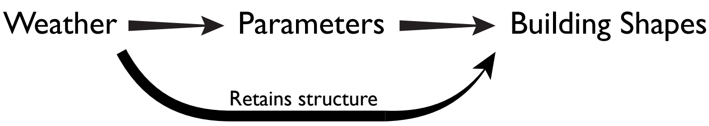
The important structure in the two spaces here is the way that the objects (model or wood) bend. Each motion of the computer model corresponds to a model of the wooden one and vice versa:



This well constructed map between different spaces, that retains the right structure, both helps in the design process and enables new forms that are practical to construct.

In this case the map is provided by nature and the spaces that use it explored by people. In the reverse a human generated algorithm can allow a selection of different forms within some given constraints. This form can then be optimised with respect to data coming from nature. The Swiss Re buidling, Foster's "Gherkin" in London is a prominent, early example of this thinking. It was designed as a parameterised model that could be varied while remaining true to the central idea of the architect. This space of potential buildings was then explored to see which were best adapted to weather conditions. This gives two maps, from the space of parameters to the space of building shapes and from that to numbers, encoding performance in the given weather conditions:

The building with optimal performance can then be selected. The process of optimisation creates a new map, from weather conditions to optimal parameters:

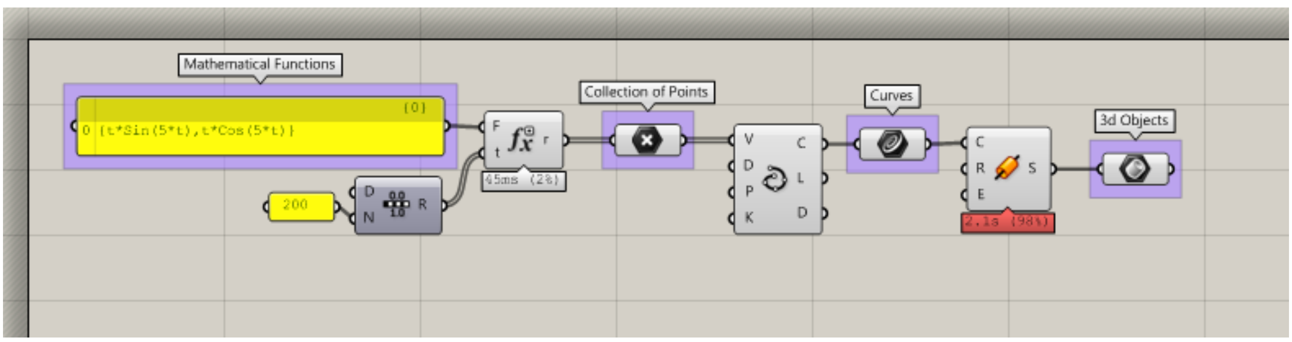


Due to the optimisation, the map from weather to building does preserve structure. With different weather conditions the shape of the building will change.

We could also break the optimisation down further as it would require creating a map from the weather to a computer model into which the building's model can be introduced. This might be a standard model taking advantage of the scientific understanding of the weather. The computer is again essential to take advantage of such knowledge. It is on the computer that the different spaces and maps involved start to multiply. Each file format defines its own abstract space and each application provides a wide variety of maps. Different formats often cover similar spaces so we must map between them, trying to ensure that the structure needed is retained. Viewed this way complex workflows can be mapped out and, if they are part of a more general project, then considered as a single map. Some of these maps might be part of a more general design project. Category theory, therefore, gives a single metaphor for often very different parts of the design process.

Using software, however, can be limiting as the only spaces and maps available on the computer are those defined by others. In order to really explore new spaces and maps requires programming. Visual programming environments such as the Grasshopper plugin for Rhino, provide an ideal way to explore these ideas. In these systems maps and spaces are considered explicitly, though the order is reversed from that given above. The boxes provide the maps and the lines joining them carry the spaces. For example consider taking a mathematical function, using this to generate a collection of points, turning those into a curve in space and then producing a three dimensional pipe. This process can be described in the diagrams given here:

The same collection of spaces and maps implemented in Grasshopper would look like this:



All these methods bring up the question of authorship. At what stage does this pass from the creator of an algorithm to the user? One way to answer this question is to consider the spaces and structures that are involved. A very simple space, such as the space of parameterized models for the Gherkin it is clear that no amount of experimentation could take authorship from the algorithm writer. On the other hand a general space such as the space of three dimensional objects that can be explored in 3d modeling software would almost always give credit to the user. There is a trickier middle ground. When people first start to understand an environment like grasshopper the designs they come up with are often quite similar to previous work. They are somehow derivative originals. The space of easily accessible opportunities opened up by the software contains certain obvious aesthetic peaks that many are drawn to. Such creations might not even have any clear author if despite being repeated they do go beyond the design intent of the software author. These questions certainly are beyond the capacity of category theory to solve, but it does give a way to consider them.

Some philosophical inspiration

The ideas considered here are grounded in perceptualism, and the notion of mathematics as a system of perception, this is modelled in Andy Goldsworthy's classic *Rivers and Tides* [RG2001], which follows simple observations deep into both metaphor and scientific models of the landscape. This leads to a notion of an applied mathematics for art, contrasted to the purer forms of much mathematical art [HSC19].

That investigation itself links back to the academic play of Alfred Jarry and the College of 'Pataphysics [Hugill12, Bok01]. Using the maps and diagrams here in the service of the rigorous subjectivity of art could provide a potential model for 'pataphysics, as the "science of the particular"[Jarry65,131]. This is a topic that delights both in defining itself and forming contradictions between those definitions. The notion of being a science of the particular is a classic example. The particular making the scientific method redundant, with its need for repetition, and thus generality. Another attempt at the appearance of a definition, however, does lead to a Category theory approach. If 'pataphysics is to metaphysics as metaphysics is to physics [Jarry65], we have an arrow from 'pataphysics to metaphysics and another map from metaphysics to physics. Yet here we are directly comparing the maps. Linking the two arrows means that we can consider a larger space of abstract worlds that contains physics, metaphysics and 'pataphysics. This space also has an arrow to itself that takes physics to metaphysics and metaphysics to 'pataphysics. We can then ask what happens to 'pataphysics? Unwinding the chain we then find a strange hierarchy:

Physics ⟵ Metaphysics ⟵ 'Pataphysics ⟵ ''Setaphysics ⟵ '''Vataphysics ⟵ ...

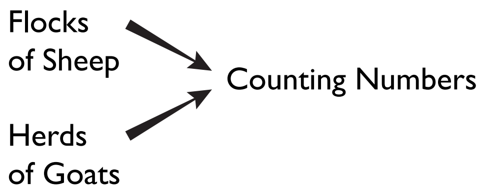
Though perhaps some would argue that this is exactly what the apostrophe at the start of 'pataphysics implies, and thus there is no where further to go. This illustrates both a strength and weakness of category theory, its ability to eat itself, considering categories (or spaces) of categories and so on, and thus disappear down a rabbit hole of abstraction.

A more serious issue is the need to understand the current world and the increasingly central role that abstraction and algorithms are taking in it. While technical mathematics is clearly playing a role here, a broader understanding of the impacts on society is also required. To contrast with the mathematics of logical positivism, Matthew Handelman describes the need (and utility) for a negative mathematics to critique and study this social impact [Handelman19]. Indeed, powerful examples of such critiques are emerging for example *Weapons of Math Destruction* by mathematician Cathy O'Neil [O'Neil16] and in mathematics education the call to rehumanise mathematics [GG18].

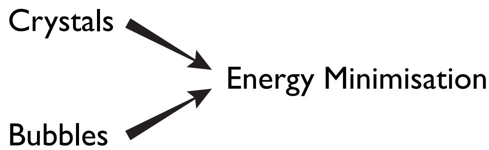
The notion of category theory itself as a system for use beyond the abstract reaches of mathematics is well established. For those wishing to develop the ideas further *Conceptual Mathematics* by William

Lawvere and Stephen Schanuel [LS98] provides a more detailed and structured approach with a general reader in mind. A more philosophical take comes in *Mathematics of the Transcendental* by Alain Badiou [Badiou14]. A lighthearted approach is taken in *How to Bake Pi* by Eugenia Cheng [Cheng16].

To finish we return to the notion of exploring simpler structures. This can have another great benefit, revealing that seemingly disparate processes are intimately linked. Numbers themselves are an example of this. We do not need separate systems for sheep, goats and coffee cups. By learning to count once we can apply the system to everything. In terms of category theory we have a map from:



This map respects structure as the operation of combined two collections to make a larger one corresponds to adding their respective counts. This idea was developed for more subtle systems by Gilles Deleuze in his concept of engineering diagrams [Deleuze94]. Manuel DeLanda gives an example by considering crystals and bubbles [Delanda00]. These are quite different objects, initially it appears that the rules governing the positions of atoms as each forms are distinct. Yet when those atomic interactions are considered in terms of energy minimisation it becomes apparent that both sets of rules share significant structure:



In the spirit of the game we describe we hope you are already thinking now about the possible maps between flocks and herds, crystals and bubbles.

For an artist or designer the ability to spot that two different systems have important similarities is an essential one. It can reduce the amount of work required and allow surprising new advances as the intuitions and ideas of each area can be translated into the other.

Bibliography

[Alberti88] Leon Battista Alberti *De re aedificatoria. On the art of building in ten books.* (translated by Joseph Rykwert, Robert Tavernor and Neil Leach) MIT Press, 1988.

[Albers65] Annie Albers *On Weaving* Wesleyan University Press 1965

[Badiou14] Alan Badiou *Mathematics of the Transcendental* Bloomsbury Academic 2014

[Bok01] Christian Bok '*Pataphysics: The Poetics of an Imaginary Science* Northwestern University Press 2001

[Carpo11] Mario Carpo *The Alphabet and the Algorithm* The MIT Press 2011

[Cheng16] Eugenia Cheng *How to Bake Pi* Basic Books 2016

[Coon16] Lynda Coon Gendering Dark Age Jesus
Gender & History, Vol.28 No.1 April 2016, pp. 8–33.

[Deleuze94] Gilles Deleuze *Difference and Repetition* (trans. Paul Patton)
Columbia University Press 1994

[Delanda00] Manuel DeLanda Deleuze, Diagrams, and the Genesis of Form
Amerikastudien / American Studies Vol. 45, No. 1, pp. 33-41 2000

[FKLMS12] Moritz Fleischmann, Jan Knippers, Julian Lienbard, Achim Menges and Simon Schleicher Material Behaviour
Architectural Design March/April 2012 pp 44-52 2012

[GG18] Imani Goffney and Rochelle Gutiérrez (Editors)
Rehumanizing Mathematics for Black, Indigenous and Latinx students.
NCTM 2018

[Grey08] Jeremy Grey *Plato's Ghost*
Princeton University Press 2008

[HSC19] Edmund Harriss, Carl Smith and Angela Carpenter
Geometry in the walnut grove: an applied mathematical approach to art.
Journal of Mathematics and the Arts Jan 2019

[Handelman19] Matthew Handelman *The Mathematical Imagination*
Fordham University Press 2019

[Hugill12] Andrew Hugill *'Pataphysics: A useless guide*
MIT Press 2012

[Jarry65] Alfred Jarry, (Roger Shattuck and Simon Taylor, Editors)
Selected Works of Alfred Jarry
Grove, New York 1965

[LS98] F. William Lawvere and Stephen H. Schanuel *Conceptual Mathematics*
Cambridge University Press 1998

[Lewis94] David Lewis *We, the Navigators: The Ancient Art of Landfinding in the Pacific*
University of Hawaii Press 1994

[MacLane10] Saunders MacLane *Categories for the Working Mathematician (2nd edition)*
Springer 2010

[Manin07] Yuri Manin *Mathematics as Metaphor (Collected Works)*
AMS 2007

[Maurus830] Hrabanus Maurus *In honorem sanctae crucis (In Honour of the Holy Cross)*
Biblioteca Apostolica Vaticana. Vatican City, Reg. lat. 124, fol. 8 verso.

[Menges12] Achim Menges Material Generation
ACADIA 12: Synthetic Digital Ecologies pp 21-24 2012

[Nargi11] Lela Nargi *Knitting Around the World:*
A Multistranded History of a Time-Honored Tradition
Voyageur Press 2011

[O'Neil16] Cathy O'Neil *Weapons of Math Destruction:*
How Big Data Increases Inequality and Threatens Democracy
Crown 2016

[PL91] Przemyslaw Prusinkiewicz, Aristid Lindenmayer, J.S. Hanan and F.D. Fracchia
The Algorithmic Beauty of Plants
Springer 1991

[RG2001] Thomas Riedelsheimer and Andy Goldsworthy *Rivers and Tides*
Documentary Film 2001

[Taruskin09] Richard Taruskin *Music from the Earliest Notations to the Sixteenth Century*
Oxford University Press 2009

[W-T45] D'arcy Wentworth Thompson *On Growth and Form*
Cambridge University Press 1945